RAN-2403000503033001

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S. Y. B. Sc. (Sem. - III) Examination March - 2025

Mathematics - MH - MJ - 303 - Bivariate Calculus

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(૧) નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S. Y. B. Sc. (Sem. - III)**

Name of the Subject :

☛ **Mathematics - MH - MJ - 303 - Bivariate Calculus**

Subject Code No.: **2403000503033001**

Seat No.:

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the corresponding question.

Q. 1. Answer the following (Any ten) **10**

1. If $x = r \cos\theta$, $y = r \sin\theta$ then find $\frac{\delta(x, y)}{\delta(r, \theta)}$.
2. Obtain U_y for $U(x, y, z) = \log(x^2 + y^2 + z^2)$.
3. Evaluate $\int_0^1 \int_2^3 dy\, dx$.
4. Define: Beta function.
5. Define Jacobian of trivariate function.
6. Evaluate $\int_0^\infty x^4 e^{-x}\, dx$.
7. Define: Minima of a Bivariate function.
8. Evaluate $\int_0^1 x^2 (1 - x)^3\, dx$.
9. Obtain $\lim_{y \to 0} \left\{ \lim_{x \to 0} \frac{x^2 + y^2}{x^2 - y^2} \right\}$.
10. Prove that $\beta(l, m) = \beta(m, l)$.

11. Find the value of $\int_0^1 \int_0^y dy\, dx$.

12. Define: Composite function.

Q. 2. Answer the following (Any two) **10**

1. If f is a differentiable function in x and y such that $xf_x + yf_y = mf$, then prove that f is a homogeneous function of degree m.
2. Discuss the continuity of the function $f(x, y)$ at the point (0, 0) where

$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}, (x, y) \neq (0, 0)$$

$$= 2, \qquad (x, y) \neq (0, 0)$$

3. If $z = f(x, y)$ and $x = e^{-u} + e^{v}$, $y = e^{u} + e^{-v}$, then prove that $\frac{\delta z}{\delta u} - \frac{\delta z}{\delta v} = y\frac{\delta z}{\delta y} - x\frac{\delta z}{\delta x}$.

Q. 3. Answer the following (any two) **10**

1. Obtain expression of e^{ax+by} in the form of powers of x and y.
2. If $u^3 + v^3 = x + y$, $u^2 + v^2 = x^3 + y^3$ then show that $\frac{\delta(u, v)}{\delta(x, y)} = \frac{1}{2}\frac{y^2 - x^2}{uv(u - v)}$
3. Find extreme values of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

Q. 4. Answer the following (any two) **10**

1. Change the order of integration of the double integral

$$\int_0^5 \int_{\frac{12y}{5}}^{\sqrt{169 - y^2}} f(x, y)\, xy\, dx.$$

2. If S is a region bounded by x-axis and the line $y = x$ and $x + y = 2$ then find the value of $\iint_S (x^2 + y^2)\, dx\, dy$.

3. Evaluate $\int_0^{\pi} \int_0^{a(1+a\cos\theta)} r^2 \sin\theta \cos\theta\, d\theta dr$

Q. 5. Answer the following (any two) **10**

1. In usual notations prove that $\beta(l, m) = \frac{\lceil l \lceil m}{\lceil l + m}$

2. Prove the following:

   i. $\beta(l, m) = 2 \int_0^{\frac{\pi}{2}} sin^{2l-1}\,\theta \,.\, cos^{2m-1}\,\theta \, d\theta$.

   ii. $\lceil n = \frac{1}{n} \int_0^{\infty} e^{-y^{\frac{1}{n}}} dy.$

3. Evaluate $\int_0^3 \frac{dx}{\sqrt{3x - x^2}}$.